2D PHOTONIC CRYSTALS ON THE ARCHIMEDEAN LATTICES
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Abstract. Results of our research on 2D Archemedean lattice photonic crystals are presented. This involves the calculations of the band structures, band-gap maps, equifrequency contours and FDTD simulations of electromagnetic propagation through the structures as well as an experimental verification of negative refraction at microwaves. The band-gap dependence on dielectric contrast is established both for dielectric rods in air and air-holes in dielectric materials. A special emphasis is placed on possibilities of negative refraction and left-handedness in these structures. Together with the familiar Archimedean lattices like square, triangular, honeycomb and Kagome’ ones, we consider also, the less known, (3²,4,3,4) (ladybug) and (3,4,6,4) (honeycomb-ring) structures.

1. Introduction

So far photonic crystals (PhC) or photonic band-gap materials have attracted a lot of attention both in fundamental and applied research [1]. The main property of PhCs is a periodically modulated index of refraction (mostly in dielectric materials). The basic concept of PhCs lies in analogy with electrons in conventional crystals. Namely, similar as for electrons in a periodic atomic potential, there are band-gaps for photons in PhCs. Since an electronic band-gap represents the main feature behind all semiconductor devices, PhC structures could provide a tool for a similar control of light. PhCs have already found many applications, such as antennas, waveguides, resonators, filters, lasers, light-emitting diodes or photonic integrated circuits. It is also worth mentioning there is a potential use of PhCs for the next generation of computers when photons can play the role of electrons in the present devices at THz or higher frequencies. Besides many applications, PhCs are the source of several interesting effects like suppression (enhancement) of spontaneous light emission, localization of light, left-handed (LH) negative refraction, right-handed (RH) negative refraction and light focusing below the diffraction limit. Band-gaps in PhCs depend on the dielectric contrast, filling factor, topology and structure symmetry.

2. Archimedean Lattices

Here we study the PhC properties on the Archimedean lattices. The Archimedean lattices shown in Fig. 1 make infinite tessellation consisting of regular convex polygons. The polygons are placed edge-to-edge to each other and are not necessarily identical. Although the Archimedean 2D lattices have been known since ancient times, Johannes Kepler was the first who gave a complete description of all 11 possible tilings as in Fig. 1, in his Harmonices Mundi (Harmony of the Worlds) in 1619 [2] (Fig. 2). The name is after Kepler’s reference to Archimedes’ regular solid polyhedra. Kepler tried to explain the distances between the planets and sun by spheres of perfect polyhedra placed inside each other as shown in Fig. 3. He associated five platonic solids with five intervals between the six then known planets, Mercury, Venus, Earth, Mars, Jupiter and Saturn.
Among the Archimedean lattices, we find the familiar square, triangular, honeycomb and Kagome structures. The first three are called regular lattices since they consist of equal polygons. So far the Archimedean lattices have been mostly studied in mathematics and statistical mechanics [3]. Therefore, proper names have not been assigned yet to all Archimedean lattices. Instead, the mathematical Grünbaum-Shephard notation is in use in terms of shape and number of polygons around each vertex, \((m_1^{n_1}, m_2^{n_2},...)\).

Fig. 1 The 11 Archimedean lattices designated using the notation of Grünbaum and Shephard.

Starting from the smallest polygon and going clockwise around a vertex, the numbers \(m_i\) denote the number of sides of each polygon, and the superscript \(n_i\) refers to the number of equal adjacent polygons. We distinguish vertices by type and species. For example the \((3^3, 4^2)\) and \((3^2, 4, 3, 4)\) lattices are of the same species but of different type.

Fig. 2 Kepler’s Harmonices Mundi published in Linz 1619.
One application of the Archimedean lattices in PhCs concerns the search for structures with nearly isotropic optical properties. Especially, the isotropy of band-gaps is a desirable property for several applications of PhCs. A number of different solutions have been suggested like quasicrystals or circular PhCs which have higher order local symmetries than the Bravais lattices. Some of the Archimedean lattices, e.g. \((3^2, 4, 3, 4)\), have also a higher order local symmetry which together with lattice periodicity could be advantageous over photonic quasicrystals.

3. Archimedean Lattice Photonic Crystals

We started with the simple Archimedean-lattice PhCs, as are square (Fig. 1i), triangular (Fig. 1j), honeycomb (Fig. 1k) and Kagome’ (Fig. 1k) lattices. Our study involved calculations of band structures and band-gap maps (plane-wave method), equifrequency contours (EFC - \(\omega(k_x, k_y)\)), FDTD (finite-difference time-domain) simulations and transmission experiments at microwaves (26-60 GHz). We were mostly interested in negative refraction and left-handedness in these PhCs. Veselago [4], in his seminal work, was the first who predicted LH materials investigating hypothetical materials which had both the electrical permittivity \(\varepsilon\) and the magnetic permeability \(\mu\) simultaneously negative. Veselago found out that the index of refraction is negative while the Poynting vector \(S\) and the wave vector \(k\) (phase velocity) are anti-parallel producing backward propagating waves. Veselago’s work has stayed just an interesting idea for more than 30 years until Pendry [5] proposed and tested the new meta-materials consisting of conducting loops (split-ring resonator (SRR)) or tubes (‘Swiss rolls’). Their magnetic permeability \(\mu\) has a resonance and a narrow frequency range with \(\mu < 0\). Soon, Shelby [6] experimentally verified negative refraction at 10. 5 GHz in the first LHM structure based on SRRs interconnected with a 2D metallic rod lattice (\(\varepsilon < 0\)).

In contrast to Veselago’s meta-materials, photonic crystals (PhC) consist usually of periodically modulated dielectric material where \(\mu > 0\). Nevertheless, it was shown that diffraction effects (\(\lambda\sim\) lattice constant) can produce effective negative refraction or even negative index and LH as in Veselago’s meta-materials [7-10]. The structures showing negative refraction can be either the LH or RH media. The losses can be much smaller in PhCs since non-conducting dielectric materials are used. Both negative refraction and LH in PhCs have been experimentally demonstrated in the microwave range [11-15]. The refraction experiments were performed on 2D dielectric and metallic PhCs by measuring the displacement of the transmitted beam at varying angle of incidence.

The necessary condition to be fulfilled for LH in PhCs is \(v_{ph} \cdot v_{gr} < 0\) or equivalently \(k \cdot S < 0\), since \(v_{gr}\) is parallel to \(S\) in large enough PhCs. The symbols \(v_{ph}, v_{gr}\) and \(S\) indicate the phase and group
velocity and the Poynting vector. We distinguish between RH and LH behavior of the wave propagation in PhCs on the sign of the $v_{ph} \cdot v_{gr}$ product.

The necessary conditions for negative refraction require that the equifrequency contours (EFCs) in PhCs are both convex (with an inward gradient) and larger than the corresponding EFCs in air. Additionally $\lambda_o \equiv c/f \geq 2 \cdot a_s$ ($a_s$ is the surface lattice constant) in order to avoid higher order Bragg diffractions out of the crystal. Negative refraction can be observed around the symmetric points in the Brillouin zone whereas LH behaviour is only possible around the $\Gamma$ point. In the case of the TM2/TE2 bands around the $\Gamma$ point, there is LH negative refraction of the Veselago type where $v_{gr}$ and $v_{ph}$ are opposite and, even LH positive refraction takes place in the TM3 band due to anisotropy [10]. It turned out that regular Archimedean lattices like square (4x4) or triangular (3x6) structures enable most effects to be observed in PhCs like negative refraction (including all-angle RH negative refraction [8]), left-handedness or superlens focusing.

**Fig. 4** FDTD simulation of the 67 GHz TE2 wave incident at 15° across the $\Gamma M$ interface of a square lattice PhC made of Al$_2$O$_3$ rods in air. The red and yellow arrows denote $v_{gr}$ and $v_{ph}$.

In a square lattice PhC made of alumina in air, negative refraction has been studied in the first two TM/TE bands. As an example, the LH negative refraction of the TE2 mode is presented in Fig. 4. and RH negative one of the TM1 wave [10, 15] in Fig. 5.

**Fig. 5** FDTD simulation of the 36 GHz TM1 wave incident at 45° across the $\Gamma M$ interface of a 2D square lattice PhC made of alumina rods in air. The red and yellow arrows denote group and phase velocity, respectively. The corresponding microwave measurements are given in the inset.
Based on the square lattice we designed the square-parquet or square-honeycomb structure [12] as shown in Fig. 6. The lattice is a 2D analog of the well known 3D woodpile lattice.

Fig. 6 Square-parquet PhC made of alumina rods in air and its three atom unit cell. The inset denotes both the measured and calculated transmission spectra.

At first sight, the structure looks as another Archimedean one but close inspection reveals that the condition of edge-to-edge placement of the adjacent polygons is not fulfilled. The PhC in Fig. 6 made of alumina rods has a large gap-to-midgap ratio of 40 %. The FDTD simulation for propagation at $\lambda/a = 4.54$ ($a$ is the near-neighbor rod distance) revealed negative refraction.

Fig. 7 26 GHz TM2 wave propagating through a honeycomb GaAs rod PhC incident at 30° across the $\Gamma K$ interface (FDTD)

In the case of hexagonal structure, we studied the Kagome lattice and honeycomb or graphite lattice (dual to triangular graph). Comparing to the triangular lattice, honeycomb and Kagome lattices have two and three atom unit cells, respectively. As a result, more complicated band structures appear. We demonstrated that both lattices (rods in air) had all-angle left-handed refraction for the TM2 band [16] where both effective indices, $n_{\text{eff}}(\omega, \theta) = \text{sgn}(\mathbf{v}_g \cdot \mathbf{k}_{\text{PhC}}) c | \mathbf{k}_{\text{PhC}} | / \omega = \pm k_{\text{PhC}} / k_{\text{air}}$ and $n_{\text{beam}}(\omega, \theta) = \sin(\theta)/\sin(\theta_i)$ ($\omega$, $\theta$, and $\theta_i$ are the frequency, incident and, refracted angle, respectively) are close to -1. In Fig. 7 the LH negative refraction in the TM2 band of the honeycomb PhC is presented.
Among irregular Archimedean lattices, we studied the \((3^2, 4, 3, 4)\) - ladybug and \((3, 4, 6, 4)\) structures. Particularly, the first one and corresponding derived non-Archimedean lattices are interesting [17-19] since they possess near and full 12-fold local rotational symmetry enabling isotropic bandgaps. The relative bandgap variations as a function of propagation direction are between 1 and 3 %. The ladybug lattice is shown in Fig. 8.

**Fig. 8** \((3^2, 4, 3, 4)\) or ladybug lattice with the four-atom unit cell cut along the \(\Gamma X\) axis.

In Fig. 9 we present the LH negative refraction in the TE3 band of a ladybug lattice PhC made of alumina \((\varepsilon = 9.6)\) rods in air.

**Fig. 9** LH negative refraction of the TM4 mode in the ladybug lattice PhC made of \(\text{Al}_2\text{O}_3\) rods in air \((\varepsilon = 9.6, \, r/a = 0.35 \text{ and } \lambda /a = 4.55)\) across the \(\Gamma X\) interface. The incident angle is \(30^\circ\). The EFCs of the TM4 band are presented in the inset.

### 4. Conclusion

In review, we described part of our research on the regular and irregular Archimedean lattice PhCs. Particularly, we investigated possibilities of negative refraction and left-handedness in these structures and for the first time LH negative refraction is demonstrated in the ladybug structure.

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